More generally, if we think of the integrand as an instantaneous rate of change, say f', then the definite integral gives the net change in the value of f. That is,

$$\int_a^b f'(t)\,dt = f(b) - f(a).$$

This is a version of the evaluation part of the FTC, and even though what we have presented is not a proof, it does give us some understanding of what the FTC is saying: **The definite integral accumulates net amount of change when the integrand is the instantaneous rate of change.**

We now return to my second goal: investigating the integral with a variable upper limit of integration. The preceding development has led us to the conclusion that

$$\int_a^b f'(t)\,dt = f(b) - f(a).$$

By replacing b with x we have

$$\int_a^x f'(t)\,dt = f(x) - f(a).$$

Notice the bonus that results. If we take the derivative of both sides, we get

$$\frac{d}{dx}\int_a^x f'(t)\,dt = f'(x).$$

(The derivative of $f(a)$ is zero since $f(a)$ is a number.) This is the antiderivative version of the FTC, in the case where the integrand is a continuous derivative. Here again, we have not presented a formal proof, but we have made significant progress in helping our students begin to understand this part of the FTC. This approach can be used to help students understand what the formulas are saying when we get into the topic of derivatives of functions defined by integrals.

The Integral Function—Class Worksheet

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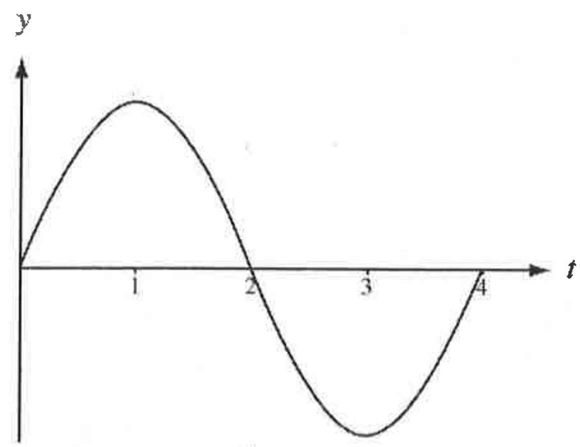
The graph of the function $y=f(t)$ is shown below. The function is defined for $0 \le t \le 4$ and has the following properties:

- The graph of f has odd symmetry around the point $(2,0)$.
- On the interval $[0,2]$, the graph of f is symmetric with respect to the line $t=1$.
- $\int_0^1 f(t)\,dt = \frac{4}{3}$.



Graph of $y=f(t)$

Special Focus: The Fundamental Theorem of Calculus

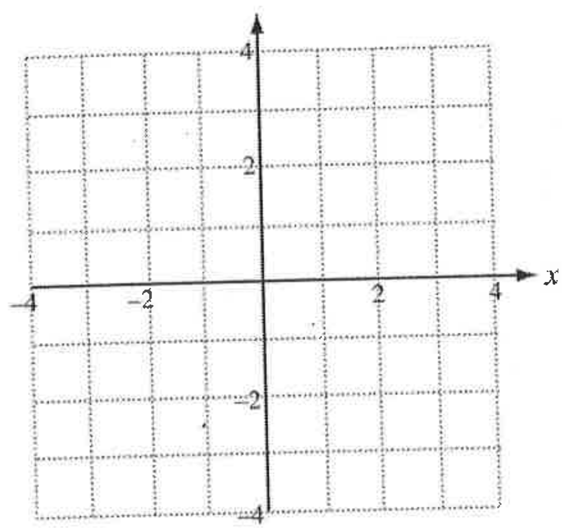
1. Let $F(x) = \int_0^x f(t)\,dt$.

   a. Complete the following table of values.

x	0	1	2	3	4
$F(x)$					

   b. Sketch your best estimate of the graph of F on the grid below.

$y = F(x)$



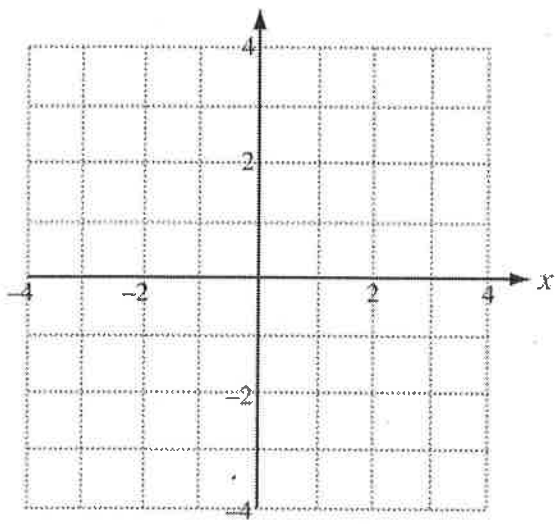
2. Let $G(x) = \int_2^x f(t)\,dt$.

a. Complete the following table of values.

x	0	1	2	3	4
$G(x)$					

b. Sketch the graph of G on the grid below.

$y = G(x)$



Special Focus: The Fundamental Theorem of Calculus

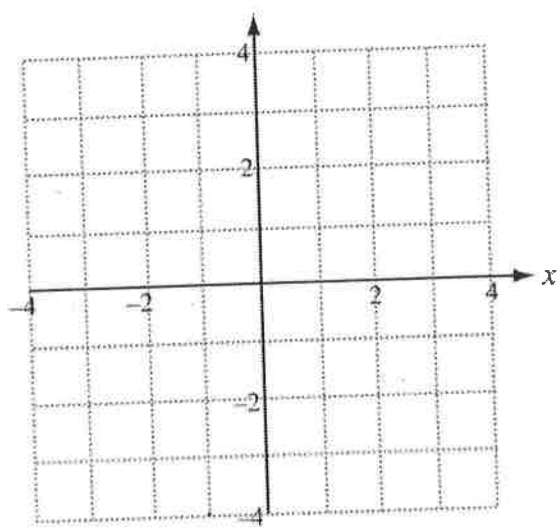
3. Let $H(x) = \int_4^x f(t)\,dt$.

   a. Complete the following table of values.

x	0	1	2	3	4
$H(x)$					

   b. Sketch the graph of H on the grid below.

$y = H(x)$



4. Complete the following table.

	$F(x)$	$G(x)$	$H(x)$
The maximum value of the function occurs at what x-value(s)?			
The minimum value of the function occurs at what x-value(s)?			
The function increases on what interval(s)?			
The function decreases on what interval(s)?			

5. Although the tables in questions 1, 2, and 3 asked only for the three functions to be evaluated at integer values of x, those functions were all continuous on the domain of $0 \le x \le 4$. Refer back to the answers you gave for function F in the table above, and explain why you believe each of these answers is correct when one considers F on its entire domain. Write your arguments in the table below. Your explanations should not rely on the graphs you sketched.

	Justification of the answers above for $F(x)$
The maximum value of the function occurs at what x-value(s)?	
The minimum value of the function occurs at what x-value(s)?	
The function increases on what interval(s)?	
The function decreases on what interval(s)?	

6. What conjectures would you make about the family of functions of the form $W(x) = \int_k^x f(t)\,dt$ for $0 \le k \le 4$, where f is the graph given at the beginning of this worksheet?